The Period of a Pendulum

Introduction

Experimental physics

In physics, theory and experiment are fundamentally intertwined; you cannot have one without the other. Experimental discoveries open new avenues for theoretical development, while at the same time new theoretical predictions motivate experimental work. Experiments are performed for various purposes. Often experiments are designed to investigate phenomena for which there exists no theoretical understanding, while other experiments are designed to test theoretical predictions. Unexpected experimental results can lead to the discovery of "new physics". Experiments are NOT done to confirm what we already know.

Many factors go into designing, building and performing an experiment. Questions arise on which the experimenter must make decisions regarding how to proceed... questions such as the following:

- How precisely must a measurement be made? (i.e. When is an experiment finished?)
- How many data points should I take? (The answer is usually not "10" nor is it "ask the instructor").
- Are my results good enough? (Think about what the goal was when you started.)

Figuring out on your own how to answer the above questions in the course of doing an experiment and the subsequent interpretation of the final result of the experiment are what we will refer to as the process of "Thinking through an experiment". Learning how to think through an experiment is one of the main goals of the lab component of this course.

Model testing

One way in which experimental physics can be done is through model testing:

- A person makes observations and collects some data.
- That person (or a different one) builds a mathematical model based on some physics which attempts to explain what's going on.
- The model is used to make predictions about a different scenario.
- The predictions of the model are tested in the new scenario to see if the predictions are correct.
  - If the data are consistent with the predictions, we say the model is supported*. New predictions are made and more testing can be done.
  - If the data are inconsistent the predictions, the model is discarded or revised.
  - If the data is ambiguous – neither convincingly in agreement or convincingly in disagreement – then either more data is needed or a different experiment should be done.

*In science, we never prove a theory. We can disprove a theory (by providing contradictory data which does not agree with the predictions of the model) or find data in support of a theory (by providing data which is consistent with the predictions), but we can never say that a theory or model is completely true. We might one day be able to use that model to make a new prediction that shows a flaw in the model.

We will get practice in model testing today by looking at the period of a simple pendulum.

Pendulum

The model we will test today is that the period $T$ of a pendulum should depend only on the length $L$ of the pendulum and the acceleration due to gravity $g$ according to the formula $T = 2\pi \sqrt{\frac{L}{g}}$. (Do not worry if you have not yet encountered pendulums in lecture; you do not need to know any theory in order to test the model.)
Our model predicts that the period should not depend on the mass of the pendulum \( m \) nor the angle \( \theta \) from which it is released. We will specifically test whether the period depends on the release angle.

### Group report template

Click on the link below to start your group report. Pick a record keeper (it should be someone different than last week) and remember to share the document with all members of the group.

[Lab 2 Template](#)

### Grading

The grading this week is identical to last week. The rubrics are repeated below for convenience.

#### In-lab rubric (4 points)

Your TA will come around at some point in the lab to speak with your group and award the in-lab points.

<table>
<thead>
<tr>
<th>Participation (2 points)</th>
<th>Lab Notebook (2 points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptable (2)</td>
<td>Unacceptable (0)</td>
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<tr>
<td>Unacceptable (0)</td>
<td>Acceptable (2)</td>
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<td>Unacceptable (0)</td>
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</tbody>
</table>

- **Acceptable (2) Participation**
  - Participates in a meaningful way in group discussions and data taking/record keeping.
  - Arrives late or leaves early.
  - Takes a superficial data set with no attempt to analyze data and improve measurements in order to leave the lab early.
  - Is disruptive or otherwise disrespectful to the group.

- **Unacceptable (0) Participation**
  - Allows partners to do most of the work.

- **Acceptable (2) Lab Notebook**
  - Keeps notebook up to date as the experiment progresses.
  - Notebook is neat, legible and organized.
  - All major elements of the experimental process are documented, including setup, procedure, notes on decisions made over course of the experiment, etc.

- **Unacceptable (0) Lab Notebook**
  - Students fill out the notebook at the end of the period.
  - Notebook entries are unintelligible.
  - Significant aspects of how the experiment was performed and what happened over the course of the experiment are omitted.

#### Report rubric (6 points)

Your TA will grade your group’s report and return it with a grade before your next lab period.

<table>
<thead>
<tr>
<th>Acceptable (2)</th>
<th>Needs Improvement (1)</th>
<th>Not Acceptable (0)</th>
</tr>
</thead>
</table>

- **Acceptable (2)**
  - Students fill out the report.

- **Needs Improvement (1)**
  - Students should improve in order to receive a grade.

- **Not Acceptable (0)**
  - Students should improve in order to receive a grade.
Activity 1: A quick statistics lesson

When conducting an experiment to test a model, we need some agreed upon criteria for determining whether or not one result is in agreement with another.

For this experiment, you will compare measured periods of a pendulum to predicted values. As we saw in last week’s experiment, however, measurements always come with some uncertainties and we cannot expect exact agreement with any prediction. You might ask, “How close is close enough?” If the predicted period is 1.0 seconds and you measure 1.5 seconds is that in agreement? What if you measure 1.05 seconds? Would that constitute agreement?

Comparing data sets

Look back at your notebook or group report from last week and find the times you measured for how long it takes a piece of paper to fall through 1 meter. In particular, find the computed average and the standard deviation.

Partner up with another group in the class and compare your values. You both were trying to answer the question "How long does it take for a piece of paper to fall 1 m?" Do your two groups agree on the answer? What does agreement mean here? What criteria do you use to determine this?

Suppose that two other students also collected the following times:

<table>
<thead>
<tr>
<th></th>
<th>Abed</th>
<th>Hazel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Times (seconds)</td>
<td>t_\text{A} = [1.09, 1.04, 0.97, 1.13, 0.95]</td>
<td>t_\text{H} = [1.37, 1.06, 1.08, 1.00, 1.15]</td>
</tr>
<tr>
<td>Average time</td>
<td>1.054 s</td>
<td>1.145 s</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.111 s</td>
<td>0.101 s</td>
</tr>
</tbody>
</table>

Is your fall time consistent with Abed's time? With Hazel's time?

Gaussian (normal) distributions

When we make observations of something which varies with random fluctuations, that data typically obeys a Gaussian (also called a Normal) distribution. Such a distribution can be characterized by parameters like the average, \( \mu \), (or mean) and standard deviation, \( \sigma \). As we saw last week, the standard deviation tells us something about the typical “spread” in values. It tells us how wide the distribution is.

Take a look at the figure below which shows a Gaussian distribution evolving over time as we add more and more data.
As the number of drops increases, the distribution looks more and more like a smooth Gaussian and we get better estimates for both the average and the standard deviation. But notice that the standard deviation (the width of the distribution) doesn’t change much; taking more data doesn’t make the distribution more narrow.

For that reason, we need to define a different parameter... one that will scale as the number of data points increases to reflect the fact that our estimate of the uncertainty also improves with the number of counts. This parameter is called the **standard deviation of the mean**, $\sigma_\mu$ (or, sometimes called the **standard error**):

$$\sigma_\mu = \frac{\sigma}{\sqrt{N}}.$$

If we look now at those data distributions again, we can see that the standard deviation of the mean does decrease as we add more data. Hence, we can now more carefully state our average and its uncertainty as $\mu \pm \sigma_\mu$. 
Using the plot with 500 drops, we can now say that the average fall time is $T = 0.997 \pm 0.006$ seconds.

**Criteria for establishing agreement – $t'$**

Now suppose we have two measurements which we want to compare: $A \pm \Delta A$ and $B \pm \Delta B$. In order to determine whether they are in agreement, we will a measure called $t'$:

$$t' = \frac{A - B}{\sqrt{(\Delta A)^2 + (\Delta B)^2}}.$$ 

In the case where only one value has an uncertainty (for example, when you want to compare a measured value $A \pm \Delta A$ to a predicted or literature value $B$), this simplifies to

$$t' = \frac{A - B}{\Delta A}.$$ 

**Agreement**

If the values are within one uncertainty of each other, it is possible that the difference is due only random chance. We will consider this to constitute agreement: $|t'| \leq 1$. (Note that agreement might turn into disagreement if more data is taken and the size of the uncertainties shrink. Remember that we can never prove a model correct... we can only say that current data supports its.)

**Disagreement**
If the values are more than three uncertainties away from each other, it is statistically unlikely that the difference is due only random chance. We will consider this to constitute disagreement: $|\Delta r| \geq 3$.

Inconclusive

If the values are between one and three uncertainties of each other, we cannot say with certainty if the difference is random chance or a real disagreement. We will consider this to be inconclusive: $1 \leq |\Delta r| \leq 3$.

Revisiting your data

Now, return to your drop data and the data from Abed, Hazel and the other group in this class. Compute the standard deviation of the mean and use the above criteria to look for agreement between your average value and that of the others. Are your results in agreement with the others? In disagreement? Inconclusive?

If you are in disagreement, what could be some reasons?

Remember to record your values and your conclusions in the group report!

Activity 2: The period of a pendulum

You have been provided with a variety of rods, clamps and other apparatus with which you can make a pendulum. Your task is to test whether or not the period of a pendulum is independent of the initial angle as predicted by the model: $T = 2\pi \sqrt{\frac{L}{g}}$. Using the apparatus provided it is possible to make very precise measurements if your experimental technique is good. Since we want you to learn how to think your way through these experiments, we are not going to specify how to best use the apparatus at your disposal. Instead it is up to you to figure out how to get the most out of the equipment in the time available to you.

Testing $\theta \leq 10^\circ$

Begin by constructing a pendulum and determine the predicted value for the period given your pendulum length. Measure the period for $\theta = 5^\circ$ and for $\theta = 10^\circ$.

Record your data and make plots using the following Jupyter notebook:

Lab Two Notebook.ipynb

Remember to update your group report as you go, including information about (and pictures of) your pendulum and technique, preliminary data or plots, and your first thoughts.

After a short period your TA will call the class together for a quick discussion, so be prepared to share those preliminary results and to discuss how you're making your measurements.

Tips

Below are some tips to get you started:

- **You are going to be making multiple measurements of periods for different angles. How are you going to ensure that each trial for a given angle starts off the same?** How will you ensure that you begin at the same angle each time? How will you make sure that the mass is released the same way each time? You may have to spend a little bit of time trying different techniques to determine an appropriate procedure. Don’t forget to document your work in your notebook, even the attempts that you decide against using.
- **How many and at what angles will you need to take data?** The only way to know the answer to this sort of question is to look at your data as you take it. Within your group, designate at least one person to record the data, compute averages, and compare with predictions from the model in real time. Your data will tell you what you need to be doing in order to accomplish the stated goal of the experiment. For example, you might try taking some quick data at a relatively small angle and at a relatively large angle and look at the results to gain insight into what range of angles are needed. Using your data like this to inform your experimental procedure is an acquired skill; no one starts off especially good at it. We cannot tell you how to do it, the only way to develop the skill is to do it yourself.
- **For a given angle, how many periods do you need to measure and average?** Again, you have to look at your data to determine how much is enough. Ultimately your goal is to make the most precise measurement possible in the time available to you.

Note that there is no "right" answer to questions like "how much data do I take?". The answer is not "I will take data at 10 angles". Ten angles may be fine, but it is also possible that you do not have time to take a sufficient number of trials at each of 10 different angles. Even professional research scientists do not have infinite time and resources to conduct a "perfect" experiment – whatever "perfect" may mean. You (as well as professional scientists) are limited by the equipment and resources available. You have a finite amount of time to produce a result. Experimental physics is an iterative process of taking some data, evaluating that data to see what needs to be improved, and making adjustments to your apparatus and technique based on these intermediate results. Keep in mind what it is that you are trying to accomplish at the end of the day (or in this case, the end of the lab period).
Feel free to ask your instructor questions and to bounce ideas off of them or other students in the lab; everyone learns from interacting with one another. The instructors are there to help guide you through the experiment, but they will not tell you exactly what angles to measure or how many measurements to take. They will not tell you if your data are "good enough". These are things you have to decide for yourself.

Testing $\theta \leq 30^\circ$

Based on the discussion with the class, extend your experiment to angles greater than 10-degrees, but no more than 30-degrees. You are free to modify your pendulum or your technique based on ideas from the class.

Remember that the TA will come around to talk to each group and to check lab notebooks, so remember to record and think about your data as you go!

Conclusions

At the end of your report you should write up a brief summary of your final conclusion. Did your experiment agree with the model and to what degree? It is important that your conclusions be supported by your data, so make the case for why you think your conclusions are correct. If there were factors which came during the experiment which you feel influenced your results in some fashion, you should comment on that. Be complete, but do not feel like you need to write a complete essay or dissertation. A couple short paragraphs should be enough.

Use this link to submit your report

Remember to log out of your Google account after you submit!