Inductance and Impedance

- **Group lab report**
- **The Long Solenoid as an Inductor.**
- **Transient Behavior of Capacitors**
- **Verify k.**
- **Report submission**

**Group lab report**

Choose a group member to be the **record keeper**. (This person should be a different person from the record keeper last week.) After creating the lab report from the template below, make sure to click the “Share” button in the upper right corner and add all group member email addresses.

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**Lab Template**

The Long Solenoid as an Inductor.

If you have not yet seen induction in lecture, reading the first paragraph of this [link](#) will be helpful.

Mutual inductance is a measure of the EMF, $\mathcal{E}$, induced in one coil, due to a changing current in a nearby but separate coil. More specifically, the EMF induced in coil 2 by a changing current in coil 1 can be given by the equation

$$\mathcal{E}_2 = -N_2 f_{\text{resonance}} dq_2 dt = -M_{12} f_{\text{resonance}} dI_1 dt \quad (1)$$

where

- $N_2$ is the number of turns of wire in coil 2
- $q_2$ is the magnetic flux in coil 2
- $M_{12}$ is the mutual inductance for the two coils
- $I_1$ is the current in coil 1

In the case of a long coil with a separate coil closely wrapped about its center, $M_{12}$ is calculable from the physical description of the coil

$$M_{12} = \mu_0 n_1 N_2 A_1 \quad (2)$$

where

- $\mu_0 = 4\pi \times 10^{-7} \text{N/A}^2$ is the permeability of free space
- $n_1$ is the number of turns per unit length of coil 1
- $N_2$ is the total number of turns on coil 2
- $A_1$ is the cross-sectional area of coil 1

If a varying current in one coil induces an EMF in any nearby coil, it must also induce an EMF in itself. By analogy, one would expect that the self-induced EMF ought to be representable by equations similar to Eqs. (1) and (2), i.e.,

$$\mathcal{E}_1 = -N_1 f_{\text{resonance}} dq_1 dt = -f_{\text{resonance}} dI_1 dt \quad (3)$$

where

- $N_1$ is the total number of turns on coil 1
- $q_1$ is the flux linking coil 1 to itself
- $L_1$ is the coefficient of self-inductance

Again, by analogy to the mutual inductance, we are tempted to guess that there will be simple coil arrangements where $L_1$ may be calculated from first principles. Since a long straight coil, generally called a solenoid, is easy to measure and to describe, we will again consider that case.

Possibly. The term is used for a long, coiled cylinder of wire in the context of Physics. Such coils are often parts of electromagnets that move objects linearly, and people have subsequently started calling such devices solenoids as well. In particular, what is called a 'starter solenoid' in cars is a device that is used to make the physical connection for current to a vehicle’s starter motor.

The flux at the center of a long solenoid is given by
Differentiating Eq. (4) and substituting into Eq. (3), we have

\[ \mathcal{E}_1 = -L_1 f_{\text{resonance}} dl_1 dt = -\mu_0 n_1 N_1 A_1 f_{\text{resonance}} dl_1 dt \]  

and therefore

\[ L_1 = \mu_0 n_1 N_1 A_1 \]  

The self-inductance of a long solenoid is directly calculable and is given by equation 6. To see the derivation of this relationship expand the section above.

When discussing solenoids the word "long" is frequently encountered. This is due to the assumption that the induced magnetic field inside the inductor is uniform, which only holds true away from the ends of the solenoid. At the end of a long solenoid, \( \varphi \) is one half of the value at the center. Thus for a finite solenoid Eq. (6) gives a value for \( L \) which is a bit too high. The correct formula is

\[ L_1 = \mu_0 n_1 N_1 A_1 K \]  

where \( K \) is a factor to correct the inductance for end effects. \( K \) depends on the ratio of the solenoid’s length to diameter, \( \ell/D \). Calculation of \( K \) is somewhat complex, so we will simply tabulate and plot the results. (See Table 1 and Fig. 1.)

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<th>( K )</th>
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**Table 1**: End effect corrections for finite length solenoids

![Figure 1](image)

From the graph it should be clear how long a "long" solenoid is from the standpoint of self inductance: a solenoid having a length to diameter ratio of 10 is already within 4% of one which is infinitely long. In precise work, the correction factor should be used.

**Transient Behavior of Capacitors**

You have seen in lecture how capacitors charge and discharge. We summarize the final results here, see below for details.
An EMF is induced in inductive circuits only when fluxes are changing with time. It is thus clear that inductances are important in determining transient behavior. You will recall that capacitors were important in a similar way, i.e., they charged and discharged relatively quickly with the rate depending on the values of \( R \) and \( C \). In the present experiment, we will first investigate the transient behavior of circuits containing inductance and resistance: LR circuits. We will find it helpful to compare the behavior of LR circuits with RC circuits.

\[ \text{Figure 2: Charging and discharging an RC circuit} \]

**Charging**

Consider the RC circuit in Fig. 2. You may recall from your studies of the charging and discharging of capacitors, if the switch S is moved to position a (so that the battery is connected across the resistor-capacitor combination) the potential across the capacitor \( V_C \) will increase according to the equation

\[ V_C = V_E \left[ 1 - e^{\left(-\frac{t}{RC}\right)} \right] \quad (8) \]

Since the sum of the voltage across the capacitor and that across the resistor must equal \( V_E \), it follows that the voltage across the resistor, \( V_R \), must be

\[ V_R = V_E e^{\left(-\frac{t}{RC}\right)} \quad (9) \]

Eq. (9) is of the form \( V_R = V_E e^{-t/\tau} \), where \( \tau \) is called the time constant. The time required for \( V_R \) to drop to \((1/\mathrm{e})\) of \( V_E \) is just

\[ \tau = RC \quad (10) \]

**Discharging**

If now the switch is moved to point b, the capacitor will discharge through the resistor and the potential across the capacitor as a function of time will be

\[ V_C = V_E e^{-t/RC} \quad (11) \]

This time, the total potential across the capacitor and the resistor must be zero. Thus, \( V_R \) is

\[ V_R = -V_E e^{-t/RC} \quad (12) \]

In all these cases, Eq. (10) is the characteristic decay time of the RC circuit.

For the RC circuit shown in figure 2 the capacitor will charge when the switch is in position a, and discharge when the switch is moved to position b.

\[ \text{Figure 2: Charging and discharging an RC circuit} \]

The time it takes for the capacitor to charge to its full voltage is given by

\[ V_C = V_E \left[ 1 - e^{\left(-\frac{t}{RC}\right)} \right] \quad (8) \]

and the time it takes to discharge is given by.
Inductors behave in a similar fashion.

\[ V_C = V_E e^{-\frac{t}{RC}} \quad (11) \]

**Figure 3: Charging and discharging an RL circuit**

Given the circuit shown in figure 3, the rate at which an inductor charges is given by,

\[ V_L = V_E e^{-\frac{R}{L}} \quad (16) \]

**Charging**

Consider now the LR circuit shown in Fig. 3. Suppose we move the switch S to position a. The potential drop across the resistor must then be \( IR \) and, by the definition of self-inductance, Eq. (3), the potential drop across the self inductance must be \( L \frac{dv}{dt} \). The sum of these must be equal to the EMF of the battery, i.e.,

\[ V_E = IR + L \frac{dv}{dt} \quad (13) \]

The solution of this equation is

\[ I = f_{resonance} V_E R \left[ 1 - e^{-\frac{R}{L}} \right] \quad (14) \]

You can easily prove that Eq. (14) is the solution of Eq. (13) by substituting both Eq. (14) and its derivative into Eq. (13).

While Eq. (14) gives the current, it does not give us the potentials so that we can compare with the transient behavior of capacitors. The transition to potentials is, however, very easy. The current, \( I \), will have to flow through the resistor, \( R \). Thus, the potential drop \( V_R \) across \( R \) will be

\[ V_R = IR = V_E \left[ 1 - e^{-\frac{R}{L}} \right] \quad (15) \]

Again, since \( V_R + V_L \) must equal \( V_E \), we have

\[ V_L = V_E e^{-\frac{R}{L}} \quad (16) \]

Note that \( V_C \) for the \( RC \) circuit has the same form as \( V_R \) for the \( RL \) circuit, and that \( V_R \) for the \( RC \) case has the same form as \( V_L \) for the \( RL \) case. Stated in another way, while the potential across the capacitor increases with time in a charging \( RC \) circuit, the potential across an inductor decreases with time in a charging \( RL \) circuit. Clearly the two are different.

**Discharging**

If now the switch is moved to point b, the left side of Eq. (13) goes to zero and the discharge solutions are

\[ V_R = V_E e^{-\frac{R}{L}} \quad (17) \]

and

\[ V_L = V_E e^{-\frac{R}{L}} \quad (18) \]
Equation (18) is of the form $V_L = V_F e^{-t/\tau}$. The time for the potential to drop to $1/e \approx 37\%$ of the maximum value is when

$$\tau = L/R \quad (19)$$

Comparison of Eq. (19) with Eq. (10) shows that there is a difference between the functional forms for the time constants of RC and RL circuits.

Inductors behave in a similar fashion.

Given the circuit shown in figure 3, the rate at which an inductor charges and discharges is given by,

$$V_L = V_F e^{-Rt/L} \quad (16)$$

Verify $k$.

You have at your disposal a long solenoid, a variable resistance box, a function generator and a scope. Your task is to figure out how to use these tools to verify whether or not the correction factor $k$ is indeed correct. The correction factor is small, take care to properly assess, record and propagate your experimental uncertainties. You will also need to think carefully about the total resistance of your circuit, it is not just the value of the resistance you dial into the R-Box. These other sources of resistance may or may not be significant so you need to know what they are and how to account for them.

Report submission

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