Coulomb's Law

Introduction

Consider two fixed point charges, and , separated by a distance . Coulomb discovered experimentally that the electrostatic force on charge due to its interaction with is given by the equation
Since $\mathbf{F}$, $\mathbf{q_1}$, and $\mathbf{q_2}$ are all defined, \( k \) becomes a constant of proportionality which must be determined experimentally. Equation (1) is now known as Coulomb's law.

In the SI system, the unit of charge

\[ q = \frac{1}{c} \text{ coulomb} \]

is the **coulomb**. It is defined as the amount of charge which passes a point in a circuit when one ampere flows for one second. Stated mathematically,

\[ q = \frac{1}{c} \text{ coulomb} \]

\[ q = \frac{1}{c} \text{ coulomb} \]

Coulomb's law is the fundamental law of electrostatics. However, when one considers the forces acting between moving charges, one finds that Coulomb's law is not the most convenient form for making calculations. The equation is, therefore, reformulated into the much more general set of equations known as Maxwell's equations. In order to simplify Maxwell's equations, it is convenient to write the constant...
in another form, i.e.,

Thus Coulomb's law can also be written as

The constant \( \varepsilon_0 \) is called the **permittivity of free space**.

Equation (3) applies to point charges. Unfortunately, in the laboratory it is not possible to work with true point charges. Therefore, we must reformulate Coulomb's law so that it applies to a practical experiment. One such practical experiment is measurement of the force between two parallel plates maintained at a known potential difference. We shall use this approach in this experiment. We must, therefore, reformulate Coulomb’s law so that it applies to this arrangement.

**Electric field**

You will recall that in mechanics we found that the gravitational force

was given by Newton’s second law,

where
is the acceleration due to gravity. The concept of a gravitational field was very useful in calculations even though it obscured the fundamental fact that a gravitational force results from the interaction of two or more masses.

By analogy to the gravitational case, Eq.(4), it should be clear that it is useful to define an electric field by the equation

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]  

Comparison with Coulomb's law, Eq.(3), shows that for a point charge

\[ E = \frac{Q}{\varepsilon_0} \frac{1}{r^2} \]

One could calculate

\[ E = \frac{1}{2\varepsilon_0} \frac{Q}{r^2} \]

for a large flat plate by setting up Eq.(6) in differential form for an element of the total charge on the plate,

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]

and then integrating over the whole plate. It is, however, more convenient to recast Coulomb's law in still another form. This relation is known as Gauss’ law. Gauss’ law uses the concept of electric flux.

**Electric flux**

The electric flux

\[ \Phi_E = \int_{S} E \cdot dA \]

through an element of surface of area

\[ \Phi_E = \int_{S} E \cdot dA \]

is defined as the dot product of the electric field strength
and the vector pointing normal to the infinitesimal area.

Mathematically, this statement is

\[ (7) \]

or, in simplified form,

\[ (8) \]

Thus

\[ \cdot \]

is the amount of flux of electric field passing through the area.
The concept of flux can perhaps be more easily understood if one considers another vector field, e.g. the velocity of a fluid. In this case, is the amount of fluid flowing through the area.

Gauss' law

Consider now a point charge and let us calculate the total flux through a closed surface surrounding that charge. For convenience we take the closed surface as a sphere centered on the charge as shown in Fig. 2.

![Electric flux from a point charge](image)

In this particular case,
are parallel. Thus

and Eq. (7) can be rewritten as

\begin{equation}
\text{(9)}
\end{equation}

Noting that

\begin{equation}
\text{(10)}
\end{equation}

is a function of

and integrating over the complete sphere, we have

\begin{equation}
\text{(11)}
\end{equation}

By symmetry, we can be sure that

\begin{equation}
\text{(12)}
\end{equation}

is constant at any particular radius. Therefore, it can be brought out in front of the integral. The integral is now simply the total surface area of the sphere. Therefore,
at radius, i.e., the scalar form of Eq. (6), we have
\begin{equation}
(12)
\end{equation}
We have proved Eq. (12) only for a spherical surface surrounding a point charge. Application of the superposition theorem and the concept of solid angle shows that the theorem applies to the total charge inside that closed surface and that the integral is zero for all charges outside that surface. Stated mathematically, Gauss' law is
\begin{equation}
(13)
\end{equation}
where
\begin{equation}
\text{(show error message)}
\end{equation}
includes only those charges inside the gaussian surface.

Electric field in a parallel plate capacitor

Consider a parallel plate capacitor consisting of two large conducting plates of area
\begin{equation}
\text{(show error message)}
\end{equation}
separated by a small distance
\begin{equation}
\text{(show error message)}
\end{equation}
The plates carry equal and opposite charge
\begin{equation}
\text{(show error message)}
\end{equation}
and we ground the negatively charged plate. Let us first calculate the electric field between the plates using Gauss’s law. As shown in Fig. 3, one end of the Gaussian “pillbox” lies inside the conductor where there is no field, while the other end lies between the plates in the field which we wish to determine.

Figure 3: Gauss’ law applied to a parallel plate capacitor

By Eq. (13),

\[
\text{(14)}
\]

Solving for

\[
\text{(15)}
\]

Note that the magnitude of the charge,

\[
\text{(show error message)}
\]

we have

\[
\text{(show error message)}
\]

Since it is easier to measure the voltage
between the plates than to measure

we need a relationship between

and

The work done in transporting a charge from the grounded plate to the positively charged plate which at potential is

, i.e.

Work is defined mechanically as force times distance,

Therefore, the relation between voltage and field is
and the relation between voltage and charge is

It is useful to rewrite Eq. (19) as

where

is called the capacitance and depends only on the geometry of the plates (i.e., their area and separation). For a parallel plate capacitor, the capacitance is given by

The unit of capacitance is the farad.

Finally, we will need to know the energy stored in a capacitor. The energy stored is the work required to place a charge onto the capacitor plates. From Eq. (21),
Therefore, 

Combining Eqs. (21) and (23), we can also write 

Force between parallel plates 
We will find the force $F$ between the plates by conducting a “thought experiment” in which the plates’ separation is decreased by an amount 

while a battery maintains a constant voltage 

between the plates. Let us see how energy is conserved in this process. 

The mechanical work performed by the plates is 

(Notice that 

is positive since
is negative and

\[ E_{\text{external}} = \frac{1}{2} C V^2 \]  

is positive.) The work done by the battery as the charge on the plates is increased by an amount

\[ W = qV \]

to maintain a constant voltage is

\[ (26) \]

However, we cannot simply equate the two terms because the energy

\[ (27) \]

of the capacitor also changes. According to Eq. (24),

\[ (28) \]

Therefore, by conservation of energy,

\[ (29) \]

so that

\[ (30) \]
From Eq. (19), the product

\[ \text{Error rendering 'mathinline' macro: (show error message).} \]

is constant at constant voltage, so that

\[ \text{(31)} \]

\[ \text{Error rendering 'mathinline' macro: (show error message).} \]

and we obtain our formula for the force between the plates,

\[ \text{(32)} \]

\[ \text{Error rendering 'mathinline' macro: (show error message).} \]

From Eq. (19), this can also be expressed as

\[ \text{(33)} \]

\[ \text{Error rendering 'mathinline' macro: (show error message).} \]

The experiment you are about to perform should now be clear. Coulomb's law, in the form of Eq. (33), predicts that the force between two parallel charged plates varies as follows:

1. directly as the square of the potential between those two plates, and
2. inversely as the square of the distance between those plates, when the potential between the plates is kept constant.

Are these predictions correct? If so, what is the value of

\[ \text{Error rendering 'mathinline' macro: (show error message).} \]

?  

Experimental Procedure

Group reports

In this course, you will be graded primarily on group lab reports. For each lab, you will receive a template file that you will flesh-out as you go along through the lab.
Select one group member to be the record-keeper. This person will be the primary person who fills out the group report, but all lab partners are expected to contribute ideas, data and arguments to the document, and everyone should agree with the final report before submission. The role of the record-keeper will change every week so that all students share in the responsibility.

Use the link below to get a copy of the file for this lab, and add your group members' names to the document. As soon as your new document opens, be sure to share it with all members of the group by selecting "Share" in the upper right corner and adding their UChicago email addresses.

Lab Template

Using Jupyter notebook

In order to do data processing and analysis, we will use Jupyter notebooks (which run on the Python programming language). Download the following notebook file:

Click here to download the Jupyter Notebook

After downloading, find the file (it should be in the "Downloads" folder) and drag it to the desktop. Rename the file something unique. Double-click on it to open. (Say "yes" or "open" to any messages that pop up). The notebook contains a combination of Python instructions as well as directions for you about what it is and how it is used.

If double-clicking didn’t work, open a command line (cmd on a PC, Terminal on a Mac) and type 'Jupyter Notebook'. Then use Jupyter's file navigator to open the downloaded worksheet.

Apparatus

The apparatus is shown schematically in Fig. 4.

![Apparatus for testing Coulomb's Law](image)

Figure 4: Apparatus for testing Coulomb's Law.

We will measure the electrostatic force between two charged aluminum plates. The upper plate is rigidly supported at a fixed height above the lab bench. The lower plate rests on the pan of an electronic scale. The scale and the lower plate may be raised or lowered with three micrometer screws. The initial plate separation is set with a ball gauge, the diameter of which you will measure with a caliper.

As we apply a potential difference, the plates become oppositely charged and an attractive force is generated. We will measure that electrostatic force as a function of potential difference and separation between the plates.

High voltage power supply

We will charge the plates with a power supply, adjustable to about 1500 volts. You will use the output labeled 20 M
The high voltage cable should be plugged into the small box on the left side of the apparatus. This box contains a 100 M resistor, which serves to limit the current to a safe value to avoid shock hazard.

**Micrometers**

Micrometers are used in this experiment to set the plate separation and determine the balance pan deflection. Adjust the micrometers to determine how they function.

How many rotations of the micrometer does it take to move micrometer end by one mm? How much does the micrometer end move if you rotate it by one smallest division?

**Calibrating the scale deflection**

In this experiment you will investigate the relationship between the force produced by an electric field between two circular plates versus the voltage applied to the plates.

**Area of the disks**

Measure the diameter of the disks with the caliper provided. Calculate the area of each disk.

**Scale pan deflection**

When we apply a potential difference to the plates, they will experience an attractive force and the lower plate will move upward slightly, giving a negative reading on the scale. Since we need to know the separation between the plates, we must determine how much that separation is affected by the added potential.

To measure this effect, remove the horizontal plastic bar which supports the upper plate. Also remove the lower plate, being careful not to damage the connecting wires.

Use the thumbscrews to secure the aluminum bar with the metric micrometer head to the plastic upright bars. Be careful not to strip the plastic threads! It may be necessary to lower the electronic scale using the three micrometers to accommodate the aluminum bar and micrometer.

**Measure the electronic scale's deflection vs. force**

Turn on the electronic scale and allow it to warm up for a minute. Press the Z (zero) button. Press lightly on the scale pan and note that the display lights and gives a positive reading. Be sure that it is set to read in grams.

Adjust the micrometer so that its tip just lightly touches the scale pan (until the scale reads non-zero). Read the micrometer and note the initial reading of the micrometer.

Next, place a 20 g mass on the scale pan to displace the pan downward slightly. Advance the micrometer until it again just touches the pan, and read the micrometer again. The difference in micrometer readings gives the displacement of the scale pan as a result of having added the 20 g mass.

Repeat this process, using 40, 60, 80, and 100-gram masses. The maximum reading you will encounter in this experiment is below 100 grams, so stay within that range.

The maximum load of the scale is 200 grams. Do not overload the scale!

Make a plot of deflection of the scale pan as a function of the load in grams.

**Setting the spacing**

Remove the aluminum bar and micrometer. Place the lower plate on the scale pan with its electrical connection facing down. Make the electrical connections shown in Fig. 4.

**Coarse adjustment**
Using micrometers A, B and C, raise the electronic scale and the lower plate until the gap is slightly larger than the ball gauge around the entire perimeter of the plates.

**Fine adjustment**

Measure the diameter of the ball gauge with a digital caliper.

Starting at a position near micrometer A, insert the ball gauge into the gap between the plates. Gently press the gauge upward onto the upper plate. Adjust micrometer A until the lower plate just touches the ball gauge. At this point the scale display should light. Similarly, move the ball gauge near micrometers B and C and adjust those micrometers in turn. It will be necessary to fine tune these adjustments two or three times to be as precise as possible in setting this initial gap spacing. **All further measurements depend on this initial setting!**

When the spacing is uniformly set, what are these initial readings of micrometers A, B and C?

**Measure the electrostatic force**

At the initial gap spacing set with the ball gauge, measure the “force” (in grams) for about six voltages spanning the full range provided by your power supply.

Repeat the above measurements for about six smaller gap spacings, down to a minimum of 0.5 mm. Think about which way and how much to turn the three micrometers. It is critical that you read and adjust the micrometers correctly!

**Data analysis**

Guided by Eq.(33), plot the force (in newtons) vs.

\[ \text{Error rendering 'mathinline' macro: (show error message).} \]

Using your earlier measurements, give a quantitative assessment of the uncertainty in both the measured force and the spacing due to scale pan displacement by the electrostatic forces.

Plotted in this manner, what functional form do you expect? Use graphing software to perform a least squares fit to your predicted functional form. What is the physical significance of the slope?

Compare your value of

\[ \text{Error rendering 'mathinline' macro: (show error message).} \]

to the literature value. Are your data consistent with Coulomb’s law to within your uncertainties?

**Assignment submission and grading**

Take a look over your report and make sure it’s complete. This lab had two smaller experiments instead of one larger one, so make sure you include enough discussion and draw sufficient conclusions on each part individually. Look back at the report rubric to make sure you’ve done a good enough job in each category.

- How did you set up your experiments and take measurements?
- Think about the questions asked throughout... how does your data help you answer those questions?
- What conclusions can you draw from your data.

If you’re all done, save your file as a PDF.

Reports are submitted digitally through an online form below. If you make a mistake, you can re-submit, but work done after the end of the lab period will **not** be accepted.

Use this link to submit your report
Each lab is worth 10 points, divided between individual in-lab participation and the group report.