Cratering

Crater formation is a complex process, and it isn’t obvious that one would be able to learn much about it from a small tabletop experiment. However, by making a few assumptions about what happens to the kinetic energy of an impactor after it strikes a surface and by applying dimensional analysis, we can come up with a simple model for how crater diameter scales with the kinetic energy of the impactor.

In this lab, you will use small impactors (steel ball bearings) on sand to explore this model and to see if your small-scale experiment can be used to estimate the kinetic energy responsible for creating the much larger Sedan Crater in Nevada.

Goals

The primary goals of this experiment are as follows:

- to learn how to evaluate experimental results;
- to learn how to answer the question "How many data points do I need?" by assessing data in real time;
- to assess both statistical and systematic uncertainties in your experiment; and
- to gain experience using Python in the Jupyter notebook environment to perform calculations, plot data, and complete least-squares fits.

Modeling crater size as a function of kinetic energy

Craters are abundant throughout the solar system. Earth’s moon and the surface of Mercury are both heavily cratered. On Earth, erosion effects tend to erase craters over geological time scales. Nevertheless, there exist numerous relatively young craters on Earth. The Chicxulub crater just off the Yucatan peninsula is one of the largest impact craters on Earth, and its creation is thought to be the cause of the mass extinction which wiped out the dinosaurs. Parts of the Nevada Test Site are covered in craters from nuclear weapons tests conducted mostly in the 1950s.

In a nutshell, craters are formed when the kinetic energy of the incoming object –
In certain cases, only one of these processes may dominate and it becomes easier to think about how a crater is formed. In such cases we can use a technique called \textit{dimensional analysis} to create a model for how crater size depends on the impactor's kinetic energy.

For this experiment, we will consider two such models.

In theory, we could use what we know about Newtonian physics to predict what would happen. In fact, if we were to examine any individual grain of sand, we could use kinematic relationships to predict where it would travel after the impact.

In practice, knowing the location and physical properties of millions of grains of sand is not feasible. And in the event that it were possible, the resulting equations would almost certainly not have any analytical solution (i.e. some equation that would predict the exact outcome for any starting configuration).

To side-step this problem, we can instead predict bulk properties of a larger system (i.e., the size of the sand crater) through other sub-disciplines of physics, such as dimensional analysis or statistical mechanics.

\textbf{Model 1: Ejection}

\begin{quote}
For the first model, we will assume that the particles which constitute the material struck by the impactor are bound loosely enough that most of the energy of the impactor goes into \textit{ejecating} material from the impact site.
\end{quote}

Assume that a spherical crater is formed by ejecting material; the size of the crater is proportional to the amount of material which was ejected. If the material has a uniform density, then the total mass of the removed material,

\[
\frac{m}{\text{mass of removed material}} = k \cdot \frac{V}{\text{volume of crater}} = k \cdot \frac{d^3}{\text{crater diameter cubed}},
\]

is proportional to the volume of the crater,

\[
\frac{m}{\text{mass of removed material}} = k \cdot \frac{V}{\text{volume of crater}} = k \cdot \frac{d^3}{\text{crater diameter cubed}},
\]

which is in turn proportional to the crater diameter cubed,

\[
\frac{m}{\text{mass of removed material}} = k \cdot \frac{V}{\text{volume of crater}} = k \cdot \frac{d^3}{\text{crater diameter cubed}},
\]

(See Fig. 1.)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{crater_geometry}
\caption{Crater geometry}
\end{figure}
At a minimum, the impactor must provide enough energy to lift the volume of mass completely out of the crater. (See Fig. 2.) If the mass is lifted to a height

\[ v \]

the kinetic energy is converted completely to a gain in potential energy of the crater material

\[ E_{\text{kin}} = E_{\text{pot}} \]

as

\[ E_{\text{kin}} = mgh \]

where

\[ g \]

is the acceleration due to gravity.

Assuming that the crater is spherical, the depth of the crater is proportional to its diameter:

\[ d = 2h \]

Using this and the mass relationship

\[ M = \frac{4}{3} \pi h^3 \]

we have

\[ h = \left( \frac{3M}{4\pi} \right)^{1/3} \]
Therefore, our first model is that the crater diameter should scale as kinetic energy to the 1/4th power:

\[ \text{Model 1: Kinetic Energy} \]

Model 2: Deformation

For the second model, we will assume that most of the energy of the impactor goes into deforming the surface by pushing the material out of the way.

Assume that a spherical crater is formed by pushing surface material out of the way; the size of the crater is proportional to the amount of material which was pushed away.

Since the material only needs to be pushed out of the way (and not raised up to some height), the energy required is simply proportional to the volume which needs to be moved:

\[ \text{Model 2: Deformation} \]

. Therefore, our second model is that the crater diameter should scale as kinetic energy to the 1/3rd power:

Grading

The grading this week is identical to last week. The rubrics are repeated below for convenience.

In-lab rubric (4 points)

Your TA will come around at some point in the lab to speak with your group and award the in-lab points.

<table>
<thead>
<tr>
<th>Participation (2 points)</th>
<th>Lab Notebook (2 points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptable (2)</td>
<td>Acceptable (2)</td>
</tr>
<tr>
<td>Unacceptable (0)</td>
<td>Unacceptable (0)</td>
</tr>
<tr>
<td>Participates in a meaningful way in group discussions and data taking/record keeping.</td>
<td>Arrives late or leaves early. Allows partners to do most of the work. Takes a superficial data set with no attempt to analyze data and improve measurements in order to leave the lab early. Is disruptive or otherwise disrespectful to the group.</td>
</tr>
</tbody>
</table>

Report rubric (6 points)

Your TA will grade your group's report and return it with a grade before your next lab period.
Experimental Procedure

<table>
<thead>
<tr>
<th>Acceptable (2)</th>
<th>Needs Improvement (1)</th>
<th>Not Acceptable (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental procedure adequately described, including diagrams as needed.</td>
<td>Some elements of the experimental procedure are omitted or unclear.</td>
<td>It is unclear how the experiment was performed.</td>
</tr>
<tr>
<td>Reasons given for decisions made regarding procedure.</td>
<td>No justification given for choices made during the experiment.</td>
<td>No uncertainties considered.</td>
</tr>
<tr>
<td>Sources of uncertainty are clearly described.</td>
<td>Important sources of uncertainty were missed.</td>
<td></td>
</tr>
<tr>
<td>Decisions regarding how uncertainties will be estimated are presented clearly.</td>
<td>No reasons presented for how uncertainties were estimated.</td>
<td></td>
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Data and analysis

<table>
<thead>
<tr>
<th>Acceptable (2)</th>
<th>Needs Improvement (1)</th>
<th>Not Acceptable (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data are clearly presented in tables and graphs.</td>
<td>Some data not recorded.</td>
<td>No data present in report.</td>
</tr>
<tr>
<td>All tables and graphs are appropriately labeled and include units.</td>
<td>Tables or graphs inadequately or incorrectly labeled.</td>
<td>Data presentation is confusing and does not relate to description of experimental procedure.</td>
</tr>
<tr>
<td>Uncertainties in data are propagated through calculations as appropriate.</td>
<td>No units assigned to measured values.</td>
<td>No treatment of experimental uncertainties.</td>
</tr>
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</table>

Conclusions

<table>
<thead>
<tr>
<th>Acceptable (2)</th>
<th>Needs Improvement (1)</th>
<th>Not Acceptable (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conclusions are clearly supported by the data.</td>
<td>Conclusions are overstated based on the data.</td>
<td>Conclusions are contradicted by the data.</td>
</tr>
<tr>
<td>Comparison of data to models or predictions include assessment of uncertainties.</td>
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Procedure

We have two potential models which are quite similar. We therefore would like to design an experiment to determine which model better describes the data. Devise an experiment that allows you to measure crater diameter as a function of impactor kinetic energy. Since your ultimate goal is to distinguish between these two similar models, you will need to think about how to achieve sufficient precision and how to collect enough data to make a conclusive statement at the end of the period.

Initial Observations

You have 5 different size steel ball bearings (impactors) and a container of fine sand of uniform grain size, along with some other pieces of equipment. Spend 5-10 minutes making some initial observations using the setup, with a focus on testing possible procedures for releasing the ball bearing and measuring craters.

Hints

Some key points to keep in mind as you consider how to go about designing and conducting your experiment are as follows:

- How can you determine the kinetic energy of the impactors (ball bearings)?
- How can you measure the diameter of the craters formed in the sand? (As a standard for defining the edge of the crater, use the highest point of the outermost ring. Note that for larger craters, the outermost ring of the crater may be relatively flat. In these cases, use the middle of the outermost ring. See Fig. 3.)
Figure 3: Determining the diameter of a crater with a ridge ring.

- How will you consistently release the impactors?
- What range of kinetic energies are necessary to test the model? (Since the model predicts a power law relationship between size and energy, you should cover at least 2 decades of energy.)

In physics, a ‘decade’ is often used to denote that something varies by a factor of 10 to some power.

For example:

- If you investigate lengths between 1 cm and 10 cm, that would be one decade

  \[
  \text{Error rendering 'mathinline' macro: (show error message).}
  \]

- If you investigate lengths between 1 cm and 1 m, that would be two decades

  \[
  \text{Error rendering 'mathinline' macro: (show error message).}
  \]

- Between 1 cm and 1 km would be a range of five decades

  \[
  \text{Error rendering 'mathinline' macro: (show error message).}
  \]

In our case, we'd like the ratio between your smallest and largest energy to be at least a factor of 100.

Record your data and make plots as you go using the following Jupyter notebook:

Lab Three Notebook.ipynb

Uncertainties

In order to either support or rule out the model under consideration, you will need to pay careful attention to the uncertainties in your measurements. For the purposes of this experiment, you will average repeated measurements and use the standard deviation of the mean (standard error) as an estimate of the uncertainty in your data.

Plotting and fitting your data

In order to distinguish between the two models, we will need to plot our data and perform a fit of the data to a generic power law function,
is our undetermined exponent and

is a proportionality constant.

Is your best-fit exponent consistent with 1/4, 1/3, neither, or both? Use the

measure introduced last week, where...

1. ...

is agreement,
2. ...

is inconclusive, and
3. ...

is disagreement.

Applying your scaling law

The function you have arrived at is an example of a scaling law. You are already familiar with at least one form of a scaling law. Aeronautical engineers construct small scale models of aircraft and test their designs in wind tunnels. If the model is aerodynamically stable, then the scaling nature of the physics involved says that the full size airplane will perform similarly. As long as the underlying assumptions remain valid, there is no reason that the functional relationship you have determined should not be valid for craters of all sizes. Craters on the moon for example should follow the same scaling law which applies to your sand craters.

Sedan Crater

Below is a Google Maps image of a portion of the Nevada Test Site where over 1000 nuclear weapons tests were conducted. You can see numerous craters formed from both above ground and below ground detonations of nuclear weapons which occurred in the 1950s. On the left side of the image is an impressive crater known as the Sedan Crater which was produced as part of Operation Plowshare to test the feasibility of using nuclear weapons for civilian construction purposes. The crater was produced by the detonation of a 104 kiloton (440 x 10^{12} J) thermonuclear explosion.
The Sedan crater has a diameter of 390 m.

Using your scaling law, what would you predict for the yield of the nuclear weapon that produced it?

This example illustrates the power of dimensional analysis and scaling laws in physics: by dropping some ball bearings into sand you were able to parameterize a mathematical function which allows you to estimate to within an order of magnitude the yield of a nuclear weapon knowing only the diameter of the crater which it produced!

Use this link to submit your report

Remember to log out of your Google account after you submit!